DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics, Semester-VI

DISCIPLINE SPECIFIC CORE COURSE – 16: ADVANCED GROUP THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Group Theory	4	3	1	0	Class XII pass with Mathematics	DSC-7: Group Theory

Learning Objectives: The objective of the course is to introduce:

- The concept of group actions.
- Sylow's Theorem and its applications to groups of various orders.
- Composition series and Jordan-Hölder theorem.

Learning Outcomes: This course will enable the students to:

- Understand the concept of group actions and their applications.
- Understand finite groups using Sylow's theorem.
- Use Sylow's theorem to determine whether a group is simple or not.
- Understand and determine if a group is solvable or not.

SYLLABUS OF DSC-16

UNIT – I: Group Actions (18 hours)

Definition and examples of group actions, Permutation representations; Centralizers and Normalizers, Stabilizers and kernels of group actions; Groups acting on themselves by left multiplication and conjugation with consequences; Cayley's theorem, Conjugacy classes, Class equation, Conjugacy in S_n, Simplicity of A_5.

UNIT – II: Sylow Theorems and Applications (15 hours)

p-groups, Sylow p-subgroups, Sylow's theorem, Applications of Sylow's theorem, Groups of order pq and p^2q (p and q both prime); Finite simple groups, Nonsimplicity tests.

UNIT – III: Solvable Groups and Composition Series (12 hours)

Solvable groups and their properties, Commutator subgroups, Nilpotent groups, Composition series, Jordan-Hölder theorem.

Essential Readings

1. Dummit, David S., & Foote, Richard M. (2004). Abstract Algebra (3rd ed.). John Wiley & Sons. Student Edition, Wiley India 2016.
2. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.
3. Beachy, John A., & Blair, William D. (2019). Abstract Algebra (4th ed.). Waveland Press.

Suggestive Readings

- Fraleigh, John B., & Brand Neal E. (2021). A First Course in Abstract Algebra (8th ed.). Pearson.
- Herstein, I. N. (1975). Topics in Algebra (2nd ed.). Wiley India. Reprint 2022.
- Rotman, Joseph J. (1995). An Introduction to the Theory of Groups (4th ed.). Springer.

DISCIPLINE SPECIFIC CORE COURSE – 17: ADVANCED LINEAR ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Linear Algebra	4	3	1	0	Class XII pass with Mathematics	DSC-4: Linear Algebra

Learning Objectives: The objective of the course is to introduce:

- Linear functionals, dual basis and the dual (or transpose) of a linear transformation.
- Diagonalization problem and Jordan canonical form for linear operators or matrices using eigenvalues.
- Inner product, norm, Cauchy-Schwarz inequality, and orthogonality on real or complex vector spaces.
- The adjoint of a linear operator with application to least squares approximation and minimal solutions to linear system.
- Characterization of self-adjoint (or normal) operators on real (or complex) spaces in terms of orthonormal bases of eigenvectors and their corresponding eigenvalues.

Learning Outcomes: This course will enable the students to:

- Understand the notion of an inner product space in a general setting and how the notion of inner products can be used to define orthogonal vectors, including to the Gram-Schmidt process to generate an orthonormal set of vectors.
- Use eigenvectors and eigenspaces to determine the diagonalizability of a linear operator.
- Find the Jordan canonical form of matrices when they are not diagonalizable.

- Learn about normal, self-adjoint, and unitary operators and their properties, including the spectral decomposition of a linear operator.
- Find the singular value decomposition of a matrix.

SYLLABUS OF DSC-17

UNIT-I: Dual Spaces, Diagonalizable Operators and Canonical Forms (18 hours)
The change of coordinate matrix; Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilators; Eigenvalues, eigenvectors, eigenspaces and the characteristic polynomial of a linear operator; Diagonalizability, Direct sum of subspaces, Invariant subspaces and the Cayley-Hamilton theorem; The Jordan canonical form and the minimal polynomial of a linear operator.

UNIT-II: Inner Product Spaces and the Adjoint of a Linear Operator (12 hours)
Inner products and norms, Orthonormal basis, Gram-Schmidt orthogonalization process, Orthogonal complements, Bessel's inequality; Adjoint of a linear operator with applications to least squares approximation and minimal solutions to systems of linear equations.

UNIT-III: Class of Operators and Their Properties (15 hours)
Normal, self-adjoint, unitary and orthogonal operators and their properties; Orthogonal projections and the spectral theorem; Singular value decomposition for matrices.

Essential Reading

1. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2019). Linear Algebra (5th ed.). Pearson Education India Reprint.

Suggestive Readings

- Hoffman, Kenneth, & Kunze, Ray Alden (1978). Linear Algebra (2nd ed.). Prentice Hall of India Pvt. Limited. Delhi. Pearson Education India Reprint, 2015.
- Lang, Serge (1987). Linear Algebra (3rd ed.). Springer.

DISCIPLINE SPECIFIC CORE COURSE – 18: COMPLEX ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Complex Analysis	4	3	0	1	Class XII pass with Mathematics	DSC-2 & 11: Real Analysis, Multivariate Calculus

Learning Objectives: The main objective of this course is to:
- Acquaint with the basic ideas of complex analysis.
- Learn complex-valued functions with visualization through relevant practicals.

- Emphasize on Cauchy's theorems, series expansions and calculation of residues.

Learning Outcomes: The accomplishment of the course will enable the students to:

- Grasp the significance of differentiability of complex-valued functions leading to the understanding of Cauchy-Riemann equations.
- Study some elementary functions and evaluate the contour integrals.
- Learn the role of Cauchy-Goursat theorem and the Cauchy integral formula.
- Expand some simple functions as their Taylor and Laurent series, classify the nature of singularities, find residues, and apply Cauchy Residue theorem to evaluate integrals.

SYLLABUS OF DSC-18

UNIT – I: Analytic and Elementary Functions (15 hours)

Functions of a complex variable and mappings, Limits, Theorems on limits, Limits involving the point at infinity, Continuity and differentiation, Cauchy-Riemann equations and examples, Sufficient conditions for differentiability, Analytic functions and their examples; Exponential, logarithmic, and trigonometric functions.

UNIT – II: Complex Integration (15 hours)

Derivatives of functions, Definite integrals of functions; Contours, Contour integrals and examples, Upper bounds for moduli of contour integrals; Antiderivatives; Cauchy-Goursat theorem; Cauchy integral formula and its extension with consequences; Liouville's theorem and the fundamental theorem of algebra.

UNIT – III: Series and Residues (15 hours)

Taylor and Laurent series with examples; Absolute and uniform convergence of power series, Integration, differentiation and uniqueness of power series; Isolated singular points, Residues, Cauchy's residue theorem, Residue at infinity; Types of isolated singular points, Residues at poles and its examples, An application to evaluate definite integrals involving sines and cosines.

Essential Reading

1. Brown, James Ward, & Churchill, Ruel V. (2014). Complex Variables and Applications (9th ed.). McGraw-Hill Education. Indian Reprint.

Suggestive Readings

- Bak, Joseph & Newman, Donald J. (2010). Complex Analysis (3rd ed.). Undergraduate Texts in Mathematics, Springer.
- Mathews, John H., & Howell, Rusell W. (2012). Complex Analysis for Mathematics and Engineering (6th ed.). Jones & Bartlett Learning. Narosa, Delhi. Indian Edition.
- Zills, Dennis G., & Shanahan, Patrick D. (2003). A First Course in Complex Analysis with Applications. Jones & Bartlett Publishers.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab:

Modeling of the following similar problems using SageMath/Python/Mathematica/Maple/ MATLAB/Maxima/ Scilab etc.

1. Make a geometric plot to show that the *n*th roots of unity are equally spaced points that lie on the unit circle $C_1(0) = \{z : |z| = 1\}$ and form the vertices of a regular polygon with *n* sides, for *n* = 4, 5, 6, 7, 8.
2. Find all the solutions of the equation $z^3 = 8i$ and represent these geometrically.
3. Write parametric equations and make a parametric plot for an ellipse centered at the origin with horizontal major axis of 4 units and vertical minor axis of 2 units.
   Show the effect of rotation of this ellipse by an angle of $\frac{\pi}{6}$ radians and shifting of the centre from (0,0) to (2,1), by making a parametric plot.
4. Show that the image of the open disk $D_1(-1-i) = \{z : |z+1+i| < 1\}$ under the linear transformation $w = f(z) = (3-4i)z + 6 + 2i$ is the open disk:
   $$D_5(-1+3i) = \{w : |w+1-3i| < 5\}.$$
5. Show that the image of the right half-plane Re $z = x > 1$ under the linear transformation $w = (-1+i)z - 2 + 3i$ is the half-plane $v > u + 7$, where $u = \text{Re}(w)$, etc. Plot the map.
6. Show that the image of the right half-plane A = $\{z: \text{Re } z \geq \frac{1}{2}\}$ under the mapping $w = f(z) = \frac{1}{z}$ is the closed disk $\overline{D_1(1)} = \{w: |w-1| \leq 1\}$ in the *w*- plane.
7. Make a plot of the vertical lines $x = a$, for $a = -1, -\frac{1}{2}, \frac{1}{2}, 1$ and the horizontal lines $y = b$, for $b = -1, -\frac{1}{2}, \frac{1}{2}, 1$. Find the plot of this grid under the mapping $f(z) = \frac{1}{z}$.
8. Find a parametrization of the polygonal path $C = C_1 + C_2 + C_3$ from $-1+i$ to $3-i$, where C_1 is the line from: $-1+i$ to -1, C_2 is the line from: -1 to $1+i$ and C_3 is the line from $1+i$ to $3-i$. Make a plot of this path.
9. Plot the line segment '*L*' joining the point $A = 0$ to $B = 2 + \frac{\pi}{4}i$ and give an exact calculation of $\int_L e^z dz$.
10. Evaluate $\int_C \frac{1}{z-2} dz$, where *C* is the upper semicircle with radius 1 centered at $z = 2$ oriented in a positive direction.
11. Show that $\int_{C_1} z dz = \int_{C_2} z dz = 4 + 2i$, where C_1 is the line segment from $-1-i$ to $3+i$ and C_2 is the portion of the parabola $x = y^2 + 2y$ joining $-1-i$ to $3+i$.
    Make plots of two contours C_1 and C_2 joining $-1-i$ to $3+i$.
12. Use the ML inequality to show that $\left|\int_C \frac{1}{z^2+1} dz\right| \leq \frac{1}{2\sqrt{5}}$, where *C* is the straight-line segment from 2 to $2+i$. While solving, represent the distance from the point *z* to the points *i* and $-i$, respectively, i.e., $|z-i|$ and $|z+i|$ on the complex plane $\mathbb{C}$.
13. Find and plot three different Laurent series representations for the function:
    $$f(z) = \frac{3}{2+z-z^2},$$ involving powers of *z*.
14. Locate the poles of $f(z) = \frac{1}{5z^4+26z^2+5}$ and specify their order.
15. Locate the zeros and poles of $g(z) = \frac{\pi\cot(\pi z)}{z^2}$ and determine their order. Also justify that Res $(g, 0) = -\pi^2/3$.